Chapter 1

Units and Unit Conversions

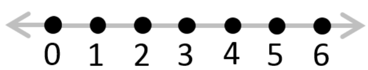
1.1 An Introduction to Vocabulary and the Order of Operations

"Don't put the cart before the horse." is an old saying that makes the point you want to get things in the right order. In mathematics, at first, this isn't easy to do. For example, to talk about operators we'll need some vocabulary but to practice vocabulary we'll need some operators. To become comfortable with the vocabulary for operations it will be a good idea to work on this section more than once.

1.1.1 The Whole Numbers

In mathematics, a **set** is a well-defined and distinct collection of objects. We use braces, $\{\ \}$,to indicate a set and a comma to separate the objects. The set of objects {1,2,3,…} where the ellipsis, (the dots …) imply the numbers continue in the same pattern forever (in this case 4, 5, 6 etc.) is known as the set of **natural numbers**. If we also include 0 in the set, we have the **whole numbers**.

The whole numbers are often visualized using the "number line",



, where the distance on the line from 0 to 1 is thought of as one "unit", the distance from 0 to 2 as two lengths of one unit and so on. The arrow to the right on the number line implies that 7, 8, 9 etc. will follow. In this course we'll spend a lot of our time combining numbers and operators.

1.1.2 Operations , Operators and Expressions

Operations transform numbers, and **operators** tells us which operation to perform. For instance, the addition operator, +, tells us to perform the operation of addition. When we see $5+15$ we know we're able to replace $5+15$ with 20.

Numbers, letters and operators are used to build a mathematical **expression** which is a meaningful collection of numbers, letters, operations and the idea of grouping. (We'll look at grouping in a few minutes.) I'll use the word **simplify** when I want you to perform all the allowable operations in an expression. Before we begin simplifying expressions though, it's important that you and I share a common vocabulary for discussing operations.

1.1.3 Some Vocabulary for Operations

We add **terms** to get a **sum**. In the expression 2 + 3, 2 is a term, 3 is a term and 5 is the sum. 2 + 3 is also a sum but the word "sum" usually refers to the single number.

We multiply **factors** to get a **product**. Three common ways to represent multiplication are $\times$ or $\bullet$ or ( ), so 2×5 or 2 • 5 or $2(5)$ all result in the product 10.

When we subtract a **subtrahend** from a **minuend** $(\text{minuend} - \text{subtrahend})$ we have a **difference**. For example, with $5-3$, 5 is the minuend, 3 is the subtrahend and the difference would be 2. We use a "dash", –, to show the operation of subtraction.

We divide a **dividend** by a **divisor** to get a **quotient** and a **remainder**. To show the operation of division we might use the $\div$ symbol ($\text{dividend} \div \text{divisor}$), a bar ($\frac{\text{dividend}}{\text{divisor}}$) or a slash, $\text{Dividend}/\text{Divisor}$. Usually we don't write remainders that happen to be 0.

One last point, if an expression can be described using two different words, the order you choose to list those words, isn't important. For instance, if we were working with $\frac{4-2}{6+1}$, and the question was, " $4-2$ is a _____ and also the _____", my answer might be difference and dividend and your answer might be dividend and difference. Both are correct.

Practice 1.1.3 Some Vocabulary for Operations

Fill in the blanks using term, sum, factor, product, minuend, subtrahend, difference, dividend, divisor or quotient.

a) Before simplifying $\frac{4-2}{6+1}$;

4 is the ____, 2 is the ____, and $4-2$ is a ____ and the ____, 6 is a ____, 1 is a ____, and $6+1$ is a ____ and the ____.

Minuend, subtrahend, difference, dividend, term, term, sum, divisor	$\Rightarrow$	Since 4 is to the left of the subtraction symbol, it's the minuend. Since 2 is to the right of the subtraction symbol, it's the subtrahend. $4-2$ is both a difference and, since it's above the division bar, it's also the dividend. 6 and 1 are being added so each is a term. $6+1$ is a sum and, since it's below the division bar, it's also the divisor.

Homework 1.1 Fill in the blanks using term, sum, factor, product, minuend, subtrahend, difference, dividend, divisor or quotient.

1) Before simplifying $2(3)$; 2 is a ____, 3 is a ____, and $2(3)$ is a ____.

2) Before simplifying $20 \div 2$; 20 is the ____, 2 is the ____ and $20 \div 2$ is a ____.

3) Before simplifying $18-12$; 18 is the ____, 12 is the ____, and $18-12$ is a ____.

4) Before simplifying $\frac{7(2)}{4-3}$; 7 is a ____, 2 is a ____ and $7(2)$ is a ____ and also the ____, 4 is the ____, 3 is the ____ and $4-3$ is both a ____ and also the ____.

1.1.4 Ordering the Four Basic Operations

If I ask someone who's good at math the best name for $\frac{4+1}{3-2}$ they'll immediately answer, "It's a quotient." That's because they've (usually unconsciously), ordered the 3 operators and realized the last operation they'll do is the division. To practice ordering operations, I'm going to ask you to count the number of operators and then name the operations using the correct order.

Before we start though, I want to make a comment about PEMDAS (Parentheses, Exponents, Multiply, Divide, Add, Subtract) or other similar memory aids that many students use to order operations. Procedures based on memory aids are too limited to handle many of the problems we'll work on this semester. So, using something like PEMDAS will be o.k. for a while, but it's also time for you to expand your tools. It takes time to understand all the subtleties when ordering operations, so we'll begin slowly. Here's our order of operations.

***Procedure* – Order of Operations**
Begin with the innermost grouping idea and work out; Explicit grouping ( ), [ ], { } Implicit grouping Operations in the dividend, in the divisor or in an exponent. 1. Start to the left and work right simplifying each operation, different from the basic four, as you come to them. 2. Start again to the left and work right simplifying each multiplication or division as you come to them. 3. Start again to the left and work right simplifying each addition or subtraction as you come to them.

Practice 1.1.4 Ordering the Four Basic Operations

Count the number of operators, name each operation using the correct order and then simplify the expression.

a) $1+2(3)+4$

There are three operators. I'll multiply first and then add left to right.

$1+6+4$	$\Rightarrow$	I started with line 2 in the procedure and multiplied. Line 2 is finished.
$7+4$	$\Rightarrow$	I moved to line 3 and began adding left to right.
11	$\Rightarrow$	I finished adding left to right.

b) $20 \div 2 - {}^{6}\!/_{2}$

There are three operators. I'll divide first left to right and then subtract.

$10-3$	$\Rightarrow$	I simplified $20 \div 2$ and then ${}^{6}\!/_{2}$. Line 2 in the procedure is finished.
7	$\Rightarrow$	I moved to line 3 and subtracted.

c) $14-10+8 \div 4 \times 2$

There are four operators. I'll multiply and divide left to right and then add and subtract left to right.

$14-10+2\times 2$ $14-10+4$	$\Rightarrow$	Moving left to right, I divided first and then multiplied. Line 2 is finished.
$4+4$ 8	$\Rightarrow$	I moved to line 3 and added and subtracted left to right.

Homework 1.1 Count the number of operators, name each operation using the correct order and then simplify the expression.

5) $15-5(3)$	6) $6/3-1+1$	7) $4(5)+3(2)-2(4)$	8) $8\times 6-8+6\div 3$
9) $24-12\div 6(2)$	10) $\left(\frac{6}{3}\right)\left(\frac{14}{2}\right)$	11) $18-14/2-7$	12) $80\div 10\times 4-2+2\times 2$

1.1.5 Explicit Grouping

In the last topic, you saw that $1+2(3)$ simplifies to 7 (and not 9). If I wanted you to add first (so the result would be 9) I'd use an explicit grouping symbol like parentheses, $(1+2)(3)$, brackets $[1+2](3)$ or braces $\{1+2\}(3)$. Operations inside grouping symbols are completed first.

We sometimes "nest" different grouping symbols so we can see what belongs together. For example, to discuss $3[12-2(4+1)]$ it's common to say, "The parentheses are **nested** inside the brackets." One last point, a number next to a grouping symbol is a factor. For example, in the expression $3[12-2(4+1)]$, both the 3 and the 2 are factors.

Practice 1.1.5 Explicit Grouping

Count the number of operators, name each operation using the correct order and then simplify the expression.

a) $(6+3)-(6-3)$

There are three operators. First, I'll complete the operations inside the parentheses left to right and then I'll subtract.

$9-3$	$\Rightarrow$	I simplified inside the parentheses left to right.
6	$\Rightarrow$	Then I subtracted.

b) $12-\left(\frac{16}{4}+2\right)$

There are three operators. Starting inside the parentheses I'll divide first and add second. The subtraction will be done last.

$12-(4+2)$ $12-6$	$\Rightarrow$	I divided and then added inside the parentheses.
6	$\Rightarrow$	To finish, I subtracted.

c) $3[12-2(4+1)]$		
There are four operators. First, I'll add inside the parentheses. Next, I'll multiply and then subtract inside the brackets. I'll finish with the product.		
$3[12-2(5)]$	$\Rightarrow$	Since the parentheses are nested inside the brackets, I began by adding.
$3[12-10]$ $3[2]$	$\Rightarrow$	Inside the brackets, I found the product first and then the difference.
6	$\Rightarrow$	Last, I multiplied to find the product.

Homework: 1.1 Count the number of operators, name each operation using the correct order and then simplify the expression.

13) $8+4-(8+4)$	14) $(15-8)(10-6)$	15) $4\{3(7-3)\}$
16) $36\div3(2+2)$	17) $8-2(5-3)$	18) $(1+3)[3(1+3)]$

1.1.6 Implicit Grouping

With implicit grouping we don't see grouping symbols, but the idea of grouping still applies. One common use for implicit grouping involves quotients. To simplify a quotient, perform all the operations in the dividend (the top) and the divisor (the bottom) before you find the quotient and remainder. For example, to simplify the quotient $\frac{18+6}{18-6}$, I'd add in the dividend $\frac{24}{18-6}$, subtract in the divisor $\frac{24}{12}$, and only then divide to get a quotient of 2.

Practice 1.1.6 Implicit Grouping

Count the number of operators, name each operation using the correct order and then simplify the expression.

a) $\frac{14-2}{2(4-2)}$		
There are four operators. First, I'll subtract in the dividend and then subtract inside the parentheses in the divisor. Next, I'll multiply in the divisor. Last, I'll find the quotient.		
$\frac{12}{2(2)}$	$\Rightarrow$	I subtracted in the dividend and then subtracted inside the parentheses in the divisor.
$\frac{12}{4}$	$\Rightarrow$	Next, I multiplied in the divisor.
3	$\Rightarrow$	To finish, I found the quotient. If you use a calculator always put the top number into the calculator first.

Homework 1.1 Count the number of operators, name each operation using the correct order and then simplify the expression.			
19) $\dfrac{6-1}{2+3}$	20) $\dfrac{(26-12)(2)}{26-12(2)}$	21) $\dfrac{2+2\times 3}{8\div 4\times 2}$	22) $\dfrac{8}{5(2)-10}$

1.1.7 More Vocabulary for Operations

Now that you've practiced the order of operations, you're able to discuss expressions in more detail. For example, should $13(2)+17$ be called a product, because of the multiplication, or a sum, because of the addition? The answer is that $13(2)+17$ is a sum because, if you follow the order of operations, the <u>last</u> operation you'd perform would be the addition.

Practice 1.1.7 More Vocabulary for Operations

Fill in the blanks using term, sum, factor, product, minuend, subtrahend, difference, dividend, divisor or quotient.

a) Before simplifying $(3+4)(11-8)$;

3 is a ____, 4 is a ____, $3+4$ is a ____ and $(3+4)$ is a ____. 11 is the ____, 8 is the ____, $11-8$ is a ____ and $(11-8)$ is a ____. Last, $(3+4)(11-8)$ is a ____.

term, term, sum, factor, minuend, subtrahend, difference, factor, product $\Rightarrow$ Since 3 and 4 are being added both are terms and $3+4$ is a sum. Enclosing the sum in parentheses, $(3+4)$, results in a factor. Since 11 is to the left of the dash it's the minuend, since 8 is to the right it's the subtrahend which makes $11-8$, a difference. Enclosing the difference in parentheses, $(11-8)$ results in a factor. Since both $(3+4)$ and $(11-8)$ are factors $(3+4)(11-8)$ is a product.

Homework 1.1 Fill in the blanks using term, sum, factor, product, minuend, subtrahend, difference, dividend, divisor or quotient.

23) Before simplifying $\dfrac{8+4}{5-2}$;

8 is a ____, 4 is a____ and $8+4$ is a ____ and also the ____. 5 is the____, 2 is the ____, $5-2$ is a ____ and the ____ and $\dfrac{8+4}{5-2}$ is a ____.

24) Before simplifying $3(5)+2(5)$;

3 is a____, 5 is a____ , $3(5)$ is a____ and a____. 2 is a____, 5 is a ____, $2(5)$ is a ____ and a ____ and $3(5)+2(5)$ is a ____.

25) Before simplifying $\left(\dfrac{6}{1}\right)\left(\dfrac{0}{3}\right)$;

6 is the____, 1 is the____, $\frac{6}{1}$ is a ____ and $\left(\frac{6}{1}\right)$ is a ____. 0 is the ____, 3 is the ____, $\frac{0}{3}$ is a____ and $\left(\frac{0}{3}\right)$ is a____. Finally, $\left(\frac{6}{1}\right)\left(\frac{0}{3}\right)$ is a ____.

26) Before simplifying $\frac{2(1+8)}{5+4}$;

1 is a ___, 8 is a ___, $1+8$ is a ___, $(1+8)$ is a ___, 2 is a____ and $2(1+8)$ is a ___ and the___. 5 is a ___, 4 is a ___, $5+4$ is a ___ and the ___ and $\frac{2(1+8)}{5+4}$ is a ___.

27) Before simplifying $3(6+4)-2(15-1)$;

6 is a ___, 4 is a ___, $6+4$ is a ___, $(6+4)$ is a ___, 3 is a ___, and $3(6+4)$ is both a ___ and the ___. 15 is the ___, 1 is the ___. $15-1$ is a ___, $(15-1)$ is a ___, 2 is a ___ and $2(15-1)$ is a ___ and the___. Last, $3(6+4)-2(15-1)$ is a ___.

28) Before simplifying $25-4(1)(2)$;

4 is a ____, 1 is a ____, 2 is a ____ and $4(1)(2)$ is both a ____ and the ____, 25 is the ____ and $25-4(1)(2)$ is a ____.

Here's some practice simplifying the expressions you worked with above.

Homework 1.1 Simplify.

29) $\frac{8+4}{5-2}$	30) $3(5)+2(5)$	31) $\left(\frac{6}{1}\right)\left(\frac{0}{3}\right)$	32) $\frac{2(1+8)}{5+4}$
33) $3(6+4)-2(15-1)$	34) $25-4(1)(2)$		

Homework 1.1 Answers

1) factor, factor, product 2) dividend, divisor, quotient 3) minuend, subtrahend, difference

4) factor, factor, product, dividend, minuend, subtrahend, difference, divisor

5) There are two operations. Multiplication is first followed by subtraction. The answer is 0.

6) There are three operations. Division is first followed by subtraction and then addition. The answer is 2.

7) There are five operations. The products are simplified first left to right, addition and subtraction left to right is last. The answer is 18.

8) There are four operations. Multiplication is first, division is next and subtraction and addition left to right is last. The answer is 42.

9) There are three operations. The division and multiplication are simplified left to right followed by the subtraction. The answer is 20.

10) Currently we think of this as three operations. The correct order of operations is to divide the 6 by the 3, multiply the result by 14 and finally divide by 2. You'll often see people divide left to right first and multiply last. As we'll see later, the results will be the same. The answer is 14.

11) There are three operations. Division is first followed by subtracting left to right. The answer is 4.

12) There are five operations. The division is first followed by the multiplications left to right. The subtraction is next and the addition last. The answer is 34.

13) There are three operations. First add inside the parentheses, then add and subtract left to right. The answer is 0.

14) There are three operations. Simplify inside each set of parentheses left to right and finally multiply. The answer is 28.

15) There are three operations. The subtraction is first followed by multiplication by 3 and finally multiplication by 4. The answer is 48.

16) There are three operations. The addition is first, the division is next and the multiplication is last. The answer is 48.

17) There are three operations. The subtraction inside the parentheses is first, the multiplication by 2 is next and the subtraction is last. The answer is 4.

18) There are four operations. The additions inside parentheses are done first left to right. Then the multiplication by 3 inside the brackets is followed by the last multiplication. The answer is 48.

19) There are three operations. First, subtract in the dividend and add in the divisor, then divide. The answer is 1.

20) There are five operations. In the dividend subtract and multiply, in the divisor multiply and subtract. Last, divide the results. The answer is 14.

21) There are five operations. In the dividend multiply first and then add. In the divisor divide and multiply left to right. The quotient is last. The answer is 2.

22) There are three operations. In the divisor multiply and subtract, then divide 8 by the result. After simplifying, the divisor becomes 0. The expression is undefined.

23) term, term, sum, dividend, minuend, subtrahend, difference, divisor, quotient

24) factor, factor, product, term, factor, factor, product, term, sum

25) dividend, divisor, quotient, factor, dividend, divisor, quotient, factor, product

26) term, term, sum, factor, factor, product, dividend, term, term, sum, divisor, quotient

27) term, term, sum, factor, factor, product, minuend, Minuend, subtrahend, difference, factor, factor, product, subtrahend, difference

28) factor, factor, factor, product subtrahend, minuend, difference

29) 4	30) 25	31) 0	32) 2	33) 2	34) 17

1.2 Multiplying and Dividing Integers

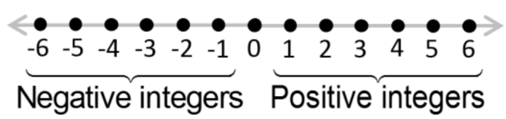
Over time, for several reasons, people found a need to move beyond the set of whole numbers. Merchants for example, realized the natural numbers easily counted the six sheep they had loaded on a ship, but, if the ship sank, the natural numbers couldn't count the loss of six sheep. To describe this loss, some merchants began to include a dash, $-$, along with the number. A natural number without the dash meant something "positive" (I have 6 sheep to sell), while the same number with a dash meant something "negative" (I've lost 6 sheep that I could have sold). In this section, we'll begin working with the integers which allows us to count both positive and negative amounts.

1.2.1 The Integers

To build the set of **integers** we begin with the natural numbers, which we rename the "positive integers". Next, we include the "negative integers" which is each natural number preceded by a dash. Last, we include 0. Zero is considered neither positive nor negative. When you're considering issues of positive or negative you're considering the **sign** of the number.

***Definition* – Integers**
The set of positive integers, negative integers and 0. Example $\{\ldots -3, -2, -1, 0, 1, 2, 3\ldots\}$

Visually, the integers run left to right from "negative infinity" (since there are an infinite number of negative integers) to "positive infinity" (since there are an infinite number of positive integers) with the negative integers to the left of zero and the positive integers to the right of zero. Notice our points, whether to the left or right of zero, are still one unit apart.



Since the natural numbers and the positive integers are the same set, we can think of each positive integer as a product that counts a quantity of 1. A logical extension for negative integers would be that they count quantities of -1. For instance, we can think of -5 as the product $5(-1)$, which counts five quantities of negative one.

1.2.2 A Note About the Dash

When you operate on integers, you'll need to wrestle with the different ways we might interpret a dash. In the previous section, we used the dash to imply the operation of subtraction. Now we're using the same symbol to imply a negative number. As you'll see in a few minutes, a couple other common uses for the dash is an implicit factor of -1 and to represent the idea of opposite (negation). In hindsight, using the same symbol for an operation and a sign might not have been the best choice, but it's what we did.

1.2.3 A Quick Look at Absolute Value

When I interview people who consistently process signs correctly, they have an unconscious (automatic) process based on memorized sign rules, and a second conscious process, based on the idea of separating issues of size from issues of sign (positive or negative). If I ask them to find the product $(-2)(-3)$, they'll use their memorized sign rules to automatically answer, "Six.". If I ask them to describe how they know the product is six most will say something like, "Well two times three is six and, since both numbers are negative, my answer's positive six." Notice they usually treat both numbers as positive to find the size of the product and then they worry about the sign.

The idea of an "absolute" value can help us with issues of size. For now, we'll think of a number's **absolute value** as it's distance from 0. This means both positive five and negative five would have an absolute value of 5 since they're both five units from 0. Notice the absolute value of a non-zero number will always be positive.

We use two vertical bars, $|\ \ |$, as the operator for the operation of absolute value. Both $|-5|$ and $|5|$ would simplify to 5. I don't want to spend much time discussing absolute value since people usually don't write and then simplify absolute value as part of their procedure for operating on signed numbers. Instead, they assume that both numbers are positive or zero and then move forward.

1.2.4 A Procedure for Multiplying Integers

In practice, your goal is to become automatic at processing signs when you multiply any two integers. Here's the procedure we'll use.

***Procedure* – Multiplying <u>Two</u> Non-Zero Integers**
1. Treat both integers as positive and find their product. 2. If both integers originally had the same sign, the product is positive. If they had different signs, the product is negative.

Notice our procedure doesn't address factors of 0. For that situation, there's a separate rule;

The Zero-Product Rule
The product of an integer and 0 can be replaced by 0. Example: $0 \times 6 = 0$

Let's practice simplifying some integer products.

Practice 1.2.4 A Procedure for Multiplying Integers *Simplify.*		
a) $(-3)(5)$		
$(3)(5)=15$	$\Rightarrow$	I thought of both integers as positive and found their product.
-15	$\Rightarrow$	The final product is negative since the original integers had different signs.
b) $(-7)(2)(-2)$		
$(-14)(-2)$	$\Rightarrow$	I began multiplying left to right. $7(2)$ is 14 and since the signs are different the product is -14.
28	$\Rightarrow$	I continued to multiply. $14(2)$ is 28 and the product is positive since the signs were the same.

Homework 1.2 Simplify.			
1) 7×-2	2) $-12(-10)$	3) $-2(-4)(-3)$	4) $(-3)(3)(-2)$
5) $-1\times-8\times0\times2$	6) $-(5\times-4)$	7) $-[-1(-5)]$	8) $3[(-2)(5)(-3)]$

1.2.5 Dividing Integers

To find the sign of a quotient, we can reuse the ideas from multiplication.

Procedure **– Dividing Two Non-Zero Integers**
1. Treat both integers as positive and find their quotient. 2. If both integers originally had the same sign, the quotient is positive, if they had different signs, the quotient is negative.

To simplify $\frac{-18}{9}$ using the procedure, I'd divide 18 by 9 to get 2, and then, since the signs are different, the final quotient would be -2.

Before starting the homework, we should discuss the issue of 0 and division. If the dividend is 0, and the divisor isn't 0, the quotient is 0. For example, $0/8=0$. If the divisor is 0, then regardless of the dividend, the quotient is undefined. So, $8/0$ is undefined.

Practice 1.2.5 Dividing Integers *Simplify.*		
a) $\frac{-12}{-3}$		
4	$\Rightarrow$	I divided 12 by 3 to get 4. The quotient's positive since the signs are the same.

b) $-\frac{8}{-2}$		
$-1\left(\frac{8}{-2}\right)$ $-1(-4)$	$\Rightarrow$	One common strategy for this situation is to think of the first dash as a factor of -1 and group the quotient using parentheses. Then 8 divided by 2 is 4 and the quotient is negative since the signs are different.
4	$\Rightarrow$	Then I found the product.

Homework 1.2 Simplify.			
9) $\frac{25}{-5}$	10) $-\frac{14}{7}$	11) $\frac{-18}{-9}$	12) $-\frac{-15}{3}$
13) $-\frac{-12}{-4}$	14) $-\frac{0}{-5}$	15) $\frac{-(-30)}{-3}$	16) $-\frac{-21}{-(-7)}$

1.2.6 Combining Operations

Let's end this section with some problems that combine multiplication and division.

Practice 1.2.6 Combining Operations		*Simplify.*
a) $\frac{(-2)(-10)}{-4}$		
$\frac{20}{-4}$	$\Rightarrow$	I must start in the dividend because it's implicitly grouped. 2 times 10 is 20 and the signs are the same, so the product is positive.
-5	$\Rightarrow$	Next, I divided 20 by 4. With different signs the quotient is negative.
b) $-\left(\frac{-30}{6}\right)\left(\frac{15}{-3}\right)$		
$-(-5)\left(\frac{15}{-3}\right)$	$\Rightarrow$	Moving left to right I simplified the first quotient. 30 divided by 6 is 5 and the signs were different, so the quotient is negative.
$-(-5)(-5)$	$\Rightarrow$	I simplified the second quotient. 15 divided by 3 is 5 and the signs were different, so the quotient is negative.
$-1(-5)(-5)$ $(5)(-5)$	$\Rightarrow$	I thought of the dash outside the first parentheses as a factor of negative one. Then, I multiplied the first two factors.
-25	$\Rightarrow$	I continued multiplying to find my final product.
c) $-\frac{16-12}{12-10}$		
$-1\left(\frac{4}{2}\right)$	$\Rightarrow$	I thought of the dash in front of the division as a factor of -1, then implicit grouping tells me to simplify the dividend and divisor first.
$-1(2)$	$\Rightarrow$	The quotient inside the parentheses simplified to 2.
-2	$\Rightarrow$	I simplified $-1(2)$ to -2.

Homework 1.2 Simplify.

17) $\dfrac{-(2\times -2)}{4}$	18) $-\left(\dfrac{-14}{2}\right)\left(\dfrac{14}{-2}\right)$	19) $\dfrac{(6)(-4)}{6-4}$	20) $-\dfrac{-2(-6)}{-4}$
21) $-\dfrac{28-14}{-14}$	22) $\dfrac{-2(-8)+8}{4(-2)}$	23) $\left(\dfrac{-9}{3}\right)\left(-\dfrac{9}{3}\right)\left(\dfrac{9}{-3}\right)$	24) $-6\times -4\div -2$
25) $-[-1(5)]+(-1)(-5)$	26) $-\dfrac{5+6}{-(18-7)}$		

Homework 1.2 Answers

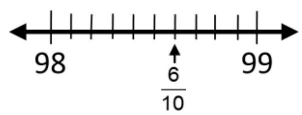
1) -14	2) 120	3) -24	4) 18	5) 0	6) 20	7) -5
8) 90	9) -5	10) -2	11) 2	12) 5	13) -3	14) 0
15) -10	16) 3	17) 1	18) -49	19) -12	20) 3	21) 1
22) -3	23) -27	24) -12	25) 10	26) 1		

1.3 Working with Fractions

The integers allow us to count something "positive", like profit and something "negative", like debt. In both cases, though, we are counting units of 1 or -1. A nice picture for this was the number line where both the negative integers and the positive integers are counting points that are an exact distance to the left or right of 0. What if we wanted to describe a distance that ended up "between" two points? For that, we'll need the idea of a rational number.

1.3.1 The Idea of a Rational Number

If your thermometer only had integers you could measure a temperature of 98° or 99° but you couldn't measure a temperature of 98.6°. To measure the temperature 98.6° we divide the distance between 98 and 99 into 10 parts of equal distance and then use 6 of the 10 parts or $\frac{6}{10}$ parts. The number $\frac{6}{10}$ is an example of a rational number.



***Definition* – Rational Number**
A number that can be written as the quotient of two integers. Example: $3/5$, -0.1, 15% Note: Rational numbers with a divisor of 0 are undefined.

Fractions are a common type of rational number. In this section, we'll begin reviewing operations with fractions. With a fraction the dividend is often called the numerator and the divisor is often called the denominator. Please notice that <u>the integers are a subset of the rational numbers</u> since every integer can be written as a fraction with a denominator of 1. For instance, the integer -3 can be written as the fraction $\frac{-3}{1}$.

1.3.2 Prime Factorization

Many procedures with fractions rely on prime factorization. To discuss prime factoring, we need a little vocabulary. A **prime number** is a natural number, greater than 1, which only has factors 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, and 17. A **composite number** is a natural number, greater than 1, which is not prime. The first few composite numbers are 4, 6, 8, 9, 10, 12, and 14.

You have prime factored a number when the number is written as the product of only prime factors. We consider $2\times2\times3$, the **prime factorization** of 12 since the factors 2 and 3 are prime. We do not consider 2×6 a prime factorization of 12 because 6 is composite. To help organize a prime factorization it's common to write the factors left to right from smallest to largest.

Practice 1.3.2 Prime Factorization

Prime factor any composite factors and then reorder the final prime factors from smallest to largest (left to right).

a) $3 \times 6 \times 2$

$3 \times 2 \times 3 \times 2$	$\Rightarrow$	I prime factored 6. (The factorization $3 \times 3 \times 2 \times 2$ is also fine.)
$2 \times 2 \times 3 \times 3$	$\Rightarrow$	I reorder the factors smallest to largest, left to right.

Homework 1.3 Prime factor any composite factors and then reorder the final prime factors from smallest to largest (left to right).

1) $5 \times 5 \times 15$	2) $4 \times 3 \times 6 \times 2$	3) $14 \times 9 \times 21$	4) $15 \times 6 \times 8$

1.3.3 Factor Rules

It's easy to see that the prime factorization of 6 is 2×3. It's harder to see that the prime factorization of 120 is $2 \times 2 \times 2 \times 3 \times 5$. The factor rules will assist in prime factoring larger numbers by helping you decide if 2, 3 or 5 are factors of your number. You should be aware that it's probably more common to call the factor rules the divisibility rules.

Factor Rules for the Natural Numbers 2, 3 and 5
2 is a factor if the number is even. Even numbers end in 0,2,4,6 or 8.
3 is a factor if the sum of the number's digits is a multiple of 3.
5 is a factor if the number ends in 5 or 0.

Practice 1.3.3 Factor Rules

Decide if the following numbers have 2, 3, or 5 as factors.

a) 12,345

2 isn't a factor	$\Rightarrow$	It's not even since it doesn't end in 0, 2, 4, 6 or 8.
3 is a factor	$\Rightarrow$	Since the sum of the digits is 15, $1+2+3+4+5=15$, and 15 has a factor of 3, 12,345 also has at least one factor of 3.
5 is a factor	$\Rightarrow$	12,345 ends in 5 so it has a factor of 5.

b) 120

2 is a factor	$\Rightarrow$	Since the number ends in 0 it's even and has 2 as a factor.
3 is a factor	$\Rightarrow$	The sum of the digits is 3, $1+2+0=3$, so 3 is a factor.
5 is a factor	$\Rightarrow$	120 has a factor of 5 since it ends in 0.

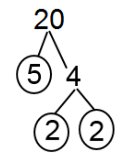
Homework 1.3 Decide if the following numbers have 2, 3, or 5 as factors.

5) 18	6) 540	7) 375	8) 119

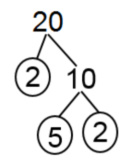
1.3.4 Factor Trees

Sometimes, a prime factorization will “pop” into your head. The factor rules, along with a factor tree, can help if nothing pops.

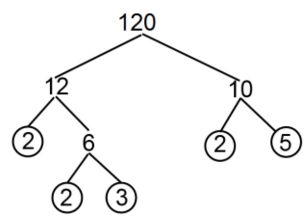
For example, to build a factor tree for 20, I might start with the factors 5 and 4. Since 5 is prime I’ll circle it and since 4 is composite I’ll continue to factor and then circle the 2’s since they’re prime. I can stop now since all the circled factors are prime. The prime factorization of 20 is $2\times2\times5$.



Notice that starting with 2 and 10 again leads to the prime factorization $2\times2\times5$. The idea that every composite number has a unique prime factorization, is often called the Fundamental Theorem of Arithmetic.



Here’s another example. When I see 120, I think of 12 times 10 so that’s how I’ll start. After continuing until only prime factors remain, I find that the prime factorization for 120 is $2\times2\times2\times3\times5$.



Sometimes finding factors of 2, 3 and 5 isn’t enough. For instance, 119 has prime factorization 7×17. Here’s a procedure that will help you find any prime factorization.

***Procedure* – To Prime Factor a Natural Number**
1. Build a factor tree using the factor rules for 2, 3, and 5.
2. After step 1 begin dividing uncircled factors by the prime numbers from 7 up to the natural number part of the number’s square root.
3. Write your prime factors in order from smallest to largest.

Homework 1.3 Prime factor the following numbers.					
9) 24	10) 42	11) 80	12) 91	13) 102	14) 429

1.3.5 Reducing Fractions

If the numerator and denominator of a fraction have a common factor, we can use the fact that any non-zero number divided by itself is 1, and the fact that the product of a number and 1 can be replaced by the number itself, to “**reduce**” the fraction.

For instance, to reduce $\frac{18}{30}$, I’d prime factor the numerator and denominator and write the prime factors from smallest to largest.

$$\frac{2\times3\times3}{2\times3\times5}$$

Next, I'd "cross out" the common factor of 2 and a common factor of 3 since each is an extra factor of 1. After reducing, I'm left with three-fifths. $\dfrac{\cancel{2}\times\cancel{3}\times 3}{\cancel{2}\times\cancel{3}\times 5}=\dfrac{3}{5}$

Notice that reducing "removes" unneeded factors of 1 from the product, it <u>doesn't</u> make the value of the fraction smaller. Here's a procedure to help reduce fractions.

Procedure **– Reducing Fractions**
1. Prime factor the numerator and denominator.
2. Reduce factors common to both the numerator and denominator.
3. Find the product of any remaining factors in the numerator and then find the product of any remaining factors in the denominator.

Practice 1.3.5 Reducing Fractions

Reduce using prime factorization.

a) $\dfrac{-36}{30}$		
$\dfrac{2\times 2\times 3\times 3}{2\times 3\times 5}$	$\Rightarrow$	I thought of both numbers as positive and I prime factored the numerator and denominator.
$\dfrac{\cancel{2}\times 2\times\cancel{3}\times 3}{\cancel{2}\times\cancel{3}\times 5}$	$\Rightarrow$	Then I reduced common factors of 1.
$-\dfrac{6}{5}$	$\Rightarrow$	Last, I multiplied the unreduced factors in the numerator and then in the denominator. Since the original signs were different, the quotient is negative.
b) $18\div 54$		
$\dfrac{2\times 3\times 3}{2\times 3\times 3\times 3}$	$\Rightarrow$	I prime factored the numerator and denominator.
$\dfrac{\cancel{2}\times\cancel{3}\times\cancel{3}}{\cancel{2}\times\cancel{3}\times\cancel{3}\times 3}=\dfrac{1}{3}$	$\Rightarrow$	Then I reduced and multiplied. I noticed I needed to include a factor of 1 in the numerator (common factors are reduced to 1, not 0) since the factor of 3 is in the denominator.

Homework 1.3 Reduce using prime factorization.

15) $\dfrac{52}{36}$	16) $-8\div 40$	17) $42/36$	18) $63\div 84$
19) $28/42$	20) $-16\div 20$	21) $-\dfrac{-51}{34}$	

1.3.6 Multiplying Fractions

Here's a fun way to multiply fractions using prime factorizations.

***Procedure* – Multiplying Fractions**
1. Multiply the prime factored form of the numerators and denominators together. 2. Reduce common factors. 3. Multiply the remaining factors in the numerator and the remaining factors in the denominator.

Practice 1.3.6 Multiplying Fractions

Multiply using prime factorization.

a) $\left(\frac{6}{15}\right)\left(\frac{25}{28}\right)$

$\frac{2\times3\times5\times5}{3\times5\times2\times2\times7}$	$\Rightarrow$	I multiplied together the prime factored form of the numerator and denominator.
$\frac{\cancel{2}\times\cancel{3}\times\cancel{5}\times5}{\cancel{3}\times\cancel{5}\times\cancel{2}\times2\times7}=\frac{5}{14}$	$\Rightarrow$	Then I reduced common factors and multiplied the remaining factors.

b) $\frac{5}{9}\times\frac{-21}{15}\times3$

$\frac{5\times3\times7\times3}{3\times3\times3\times5}$	$\Rightarrow$	I thought of all the numbers as positive and multiplied the prime factored form of the numerator and denominator. I thought of 3 as $\frac{3}{1}$.
$\frac{\cancel{5}\times\cancel{3}\times7\times\cancel{3}}{\cancel{3}\times\cancel{3}\times3\times\cancel{5}}$ $-\frac{7}{3}$	$\Rightarrow$	To finish I reduced common factors and multiplied. The final fraction is negative because I originally had one negative factor and two positive factors.

c) $\left(-\frac{14}{3}\right)\left(-\frac{9}{7}\right)$

$\frac{2\times\cancel{7}\times\cancel{3}\times3}{\cancel{3}\times\cancel{7}}$ 6	$\Rightarrow$	I multiplied, reduced common factors and multiplied the remaining factors. I noticed the denominator reduced to 1. The product is positive since the original signs were the same.

Homework 1.3 Multiply using prime factorization.

22) $-\frac{3}{4}\left(\frac{12}{15}\right)$ 23) $\left(\frac{12}{25}\right)\left(\frac{15}{9}\right)$ 24) $\frac{-5}{24}\times18$ 25) $7\left(\frac{10}{21}\right)(-6)$

26) $\frac{-4}{9}\times\frac{6}{5}\times\frac{-15}{18}$ 27) $-\frac{15}{4}\times\frac{1}{5}\times-\frac{13}{9}$ 28) $\left(-\frac{21}{4}\right)\left(\frac{2}{-9}\right)\left(\frac{3}{5}\right)\left(\frac{-6}{15}\right)$

Homework 1.3 Answers

1) $3\times5\times5\times5$ 2) $2\times2\times2\times2\times3\times3$ 3) $2\times3\times3\times3\times7\times7$ 4) $2\times2\times2\times2\times3\times3\times5$

5) 2 and 3 are factors. 6) 2, 3, and 5 are factors. 7) 3 and 5 are factors.

8) All three are **not** factors. 9) $2\times2\times2\times3$ 10) $2\times3\times7$ 11) $2\times2\times2\times2\times5$

12) 7×13 13) $2\times3\times17$ 14) $3\times11\times13$ 15) $\frac{\cancel{2}\times\cancel{2}\times13}{\cancel{2}\times\cancel{2}\times9}=\frac{13}{9}$

16) $\frac{-1\times\cancel{2}\times\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times5}=-\frac{1}{5}$ 17) $\frac{\cancel{2}\times\cancel{3}\times7}{\cancel{2}\times2\times\cancel{3}\times3}=\frac{7}{6}$ 18) $\frac{\cancel{3}\times3\times\cancel{7}}{2\times2\times\cancel{3}\times\cancel{7}}=\frac{3}{4}$ 19) $\frac{\cancel{2}\times2\times\cancel{7}}{\cancel{2}\times3\times\cancel{7}}=\frac{2}{3}$

20) $\frac{-1\times\cancel{2}\times\cancel{2}\times2\times2}{\cancel{2}\times\cancel{2}\times5}=-\frac{4}{5}$ 21) $-\frac{-3\times\cancel{17}}{2\times\cancel{17}}=\frac{3}{2}$ 22) $\frac{-1\times\cancel{3}\times\cancel{2}\times\cancel{2}\times3}{\cancel{2}\times\cancel{2}\times\cancel{3}\times5}=-\frac{3}{5}$

23) $\frac{2\times2\times\cancel{3}\times\cancel{3}\times\cancel{5}}{\cancel{5}\times5\times\cancel{3}\times\cancel{3}}=\frac{4}{5}$ 24) $\frac{-1\times5\times\cancel{2}\times\cancel{3}\times3}{\cancel{2}\times2\times2\times\cancel{3}}=-\frac{15}{4}$ 25) $\frac{\cancel{7}\times2\times5\times-1\times2\times\cancel{3}}{\cancel{7}\times\cancel{3}}=-20$

26) $\frac{\cancel{-1}\times\cancel{2}\times2\times2\times\cancel{3}\times\cancel{-1}\times\cancel{3}\times\cancel{5}}{\cancel{3}\times\cancel{3}\times\cancel{5}\times\cancel{2}\times3\times3}=\frac{4}{9}$ 27) $\frac{\cancel{-1}\times\cancel{3}\times\cancel{5}\times\cancel{-1}\times13}{2\times2\times\cancel{5}\times\cancel{3}\times3}=\frac{13}{12}$

28) $\frac{\cancel{-1}\times\cancel{3}\times7\times\cancel{2}\times\cancel{3}\times-1\times\cancel{2}\times\cancel{3}}{\cancel{2}\times\cancel{2}\times\cancel{-1}\times\cancel{3}\times\cancel{3}\times5\times\cancel{3}\times5}=\frac{-7}{25}$

1.4 An Introduction to Units and Rates

In this section we'll begin applying operations.

1.4.1 The Idea of a Unit

When we apply rational numbers we're usually measuring or counting something. If we're measuring it's common to use a "standard" unit. For example, we might use the unit foot to measure distance or the unit hour to measure time. When we're counting it's common to use an item. For instance, when we say 200 tickets the item is one ticket. Although a unit like feet or hours is often considered different from an item like tickets or people in practice both are often referred to as the unit so that's the point of view we'll take from now on.

A good first step with applied problems is recognizing any units. Here's some practice.

Practice 1.4.1 The Idea of a Unit *For each problem decide on any units.*
a) There are 38 deer in the 70-acre park.
The units are acres and deer.
b) One investment started with $1,800 and is growing by $90 a month.
The units are dollars and months

Homework 1.4 For each problem determine any units.
1) After washing 11 cars a high school car wash has earned a profit of $66.
2) After 20 minutes 18 people have voted.
3) After dieting for 12 weeks a person lost 8 pounds.
4) 85 fish were captured from 5 different lakes.

1.4.2 An Introduction to Rates

If we build a quotient, where the numerator and denominator have different units, we have a rate. For example, $\frac{50\text{ miles}}{1\text{ hour}}$ is a rate that compares time and distance. If our original denominator isn't a 1, it often helps if we divide and make it a 1. For instance instead of using $\frac{884\text{ miles}}{17\text{ gallons}}$ to describe the miles per gallon for a Toyota Prius, I think we'd all prefer to divide 884 by 17 and work with the equivalent rate $\frac{52\text{ miles}}{1\text{ gallon}}$ which tells us a Prius can go 52 miles per 1 gallon of gas.

If a rate is negative, we usually think of the numerator as negative which makes the denominator positive. For instance, if we start with, $\frac{10\text{ pounds}}{-25\text{ weeks}}$, then after dividing 10 by 25 to get 0.4, and moving the factor of -1 to the numerator, $\dfrac{-0.4\text{ pounds}}{1\text{ week}}$, we see that someone is losing, on average, four-tenths of a pound per week.

Practice 1.4.2 An Introduction to Rates

Discuss the meaning of the rate. If it's helpful, round your answer.

a) $\dfrac{\$1{,}000}{200\text{ tickets}}$

Each ticket is five dollars.	$\Rightarrow$	After dividing 1,000 by 200 I get, $\dfrac{\$5}{1\text{ ticket}}$ and I know the dollar amount for a single ticket.

b) $\dfrac{675\text{ cars}}{-4.5\text{ minutes}}$

There are 150 fewer cars per minute.	$\Rightarrow$	After dividing and making the numerator negative, $\dfrac{-150\text{ cars}}{1\text{ minute}}$, I see the number of cars is dropping by 150 every minute.

c) $-\dfrac{118\text{ cases of Polio}}{3\text{ years}}$

Polio is declining by about 39 cases per year.	$\Rightarrow$	After dividing, making the numerator negative and rounding, $\dfrac{-39.\overline{3}}{1\text{ year}} \approx \dfrac{-39}{1\text{ year}}$, I can see the cases per year.

Homework 1.4 Discuss the meaning of the rate. If it's helpful, round your answer.

5) $\dfrac{25\text{ feet}}{4.5\text{ minutes}}$	6) $-\dfrac{300\text{ coupons}}{60\text{ people}}$	7) $\dfrac{2\text{ ducks}}{25\text{ nails}}$
8) $\dfrac{1\text{ mile}}{4\text{ minutes}}$	9) $\dfrac{-\$81}{18\text{ pounds}}$	10) $\dfrac{7\text{ pounds}}{-4\text{ months}}$

1.4.3 Setting Up a "Unit" Fraction

Before money became common, people would often "buy" and "sell" using a barter system. Under a barter system a good or service would be exchanged directly for another good or service. For instance, a person who had eggs but wanted a field ploughed would try to find a second person who was willing to plough a field in exchange for eggs.

One problem with barter is that all parties must agree on the worth of an item. Let's imagine that everyone in town has gotten together and agreed on the exchange rates listed in the following table. The two headed arrow implies you should think of the relationship from left to right and from right to left.

Conversion Table for Our Town		
1 goose	⇔	5 ducks
2 ducks	⇔	25 nails
1 chicken	⇔	10 nails
1 pair of shoes	⇔	8 ducks
2 geese	⇔	15 potatoes

If someone wanted to make a trade that was listed directly in the table, things were easy. For instance, if I wanted to trade one goose for ducks, the first row tells me I should get five ducks for my one goose. What if I wanted to trade three geese for ducks? Unfortunately, this transaction isn't directly listed in the table. To complete this transaction, I'd use a unit fraction.

A **unit fraction** is a fraction, equal to 1, where you consider both the numbers and the units. For example, the fraction $\frac{1}{5}$ is obviously not equal to 1, but if I include the units in our table for geese and ducks $\frac{1 \text{ goose}}{5 \text{ ducks}}$ the fraction is equal to 1 since both the numerator and denominator are considered to have the same worth. We can use unit fractions to help with trades that are not directly listed in our conversion table. Here's a procedure that will help.

***Procedure* – Setting Up a Unit Fraction**

1. Write your original amount as a fraction with a denominator of 1.
2. Multiply your original amount by a fraction using **only** units. The denominator of your fraction should have the unit you began with and the numerator should have the unit you wish to end with.
3. Reduce units. Make sure only the unit you want to end with is left.
4. Use the conversion table to include numbers so your fraction becomes a unit fraction.
5. Multiply your two fractions together. Include the final unit in your answer.

Earlier I mentioned someone with 3 geese wanted to trade for ducks. Let's see how the procedure helps makes this happen.

Practice 1.4.3 Setting Up a Unit Fraction

Complete the transaction using a unit fraction.

a) How many ducks should I get for 3 geese?

$\frac{3 \text{ geese}}{1}$ ⇒ I wrote the original amount as a fraction with a denominator of 1.

$\frac{3 \text{ geese}}{1}\left(\frac{\text{ducks}}{\text{geese}}\right) \Rightarrow$	Next, I built a fraction using only units. I made sure the denominator of my new fraction had the same unit as the numerator of my original fraction and the unit I want to end with was in the numerator of my new fraction.
$\frac{3 \cancel{\text{geese}}}{1}\left(\frac{\text{ducks}}{\cancel{\text{goose}}}\right) \Rightarrow$	I reduced units. Since the unit ducks (the unit I want to end with) is the only unit left, I'll move on and supply values from the conversion table.
$\frac{3 \cancel{\text{geese}}}{1}\left(\frac{5 \text{ ducks}}{1 \cancel{\text{goose}}}\right) \Rightarrow$	I used the numbers from the conversion table to make the fraction a unit fraction.
$\frac{3 \cancel{\text{geese}}}{1}\left(\frac{5 \text{ ducks}}{1 \cancel{\text{goose}}}\right) = \frac{15 \text{ ducks}}{1} \Rightarrow$	I multiplied the fractions. 3 geese are worth 15 ducks.

Homework 1.4 Complete the transaction using a unit fraction.
11) How many chickens should I get for 40 nails?
12) If you trade two pairs of shoes how many ducks should you get?
13) Fourteen geese should get you how many potatoes?
14) Approximately how many nails should you get for 7 ducks?
15) 75 potatoes should get about how many geese?
16) How many pairs of shoes can you get for 24 ducks?

1.4.4 Using Multiple Unit Fractions

Sometimes, there's no direct way to trade one item for another. For instance, if I'd like to trade my potatoes for ducks, the table doesn't give me a way to do that directly. Notice though there is a way to go from potatoes to geese and that there's also a way to go from geese to ducks. Here's how to put multiple unit fractions together when no direct option is available.

***Procedure* – Using Multiple Unit Fractions**
1. Write your original amount as a fraction with a denominator of 1. 2. Begin building fractions, one at a time, always making the unit in the denominator of your new fraction the same as the unit in the numerator of your previous fraction. Continue until, after reducing units, only the unit you wish to end with is left in the numerator. 3. Use the conversion table to make your fractions into unit fractions. 4. Multiply your fractions together. Include the final unit in your answer.

Practice 1.4.4 Using Multiple Unit Fractions
Complete the transaction using unit fractions.

a) How many ducks should I get for 30 potatoes?

$\dfrac{30 \text{ potatoes}}{1}$	$\Rightarrow$ I wrote the original amount as a fraction with a denominator of 1.
$\dfrac{30 \cancel{\text{ potatoes}}}{1}\left(\dfrac{\text{geese}}{\cancel{\text{potatoes}}}\right)$	$\Rightarrow$ Next, I built a fraction using just units. I made sure the unit in the denominator, potatoes, was the same as the previous numerator unit. Unfortunately, after reducing the unit potatoes, I have geese left, not ducks.
$\dfrac{30 \cancel{\text{ potatoes}}}{1}\left(\dfrac{\text{geese}}{\cancel{\text{potatoes}}}\right)\left(\dfrac{\text{ducks}}{\text{geese}}\right)$	$\Rightarrow$ I built a second fraction using units. I made sure the unit in my new denominator, geese, was the same as the previous numerator unit.
$\dfrac{30 \cancel{\text{ potatoes}}}{1}\left(\dfrac{\cancel{\text{geese}}}{\cancel{\text{potatoes}}}\right)\left(\dfrac{\text{ducks}}{\cancel{\text{geese}}}\right)$	$\Rightarrow$ After reducing the unit geese, only the unit ducks is left. It's time to supply numbers from the conversion table.
$\dfrac{30 \cancel{\text{ potatoes}}}{1}\left(\dfrac{2 \cancel{\text{geese}}}{15 \cancel{\text{potatoes}}}\right)\left(\dfrac{5 \text{ ducks}}{1 \cancel{\text{goose}}}\right) = \dfrac{30 \times 2 \times 5}{1 \times 15 \times 1} \text{ ducks}$ 20 ducks	$\Rightarrow$ After using the table to supply my values I multiplied and reduced. I found that 30 potatoes will buy 20 ducks.

Homework 1.4 Complete the transaction using unit fractions.

17) How many ducks can I get for 12 potatoes?

18) I have 100 nails and I'd like to get a pair of shoes. Is this possible?

19) What is a fair number of chickens for 8 ducks?

20) 20 chickens will buy how many pairs of shoes?

21) 36 chickens will buy how many potatoes?

22) I have 18 potatoes about how many chickens can I get?

23) How many chickens should I get for my pair of geese?

24) I have a pair of shoes, but I need potatoes. How many potatoes should I get?

Homework 1.4 Answers

1) The units are cars and dollars.

2) The units are minutes and people.

3) The units are weeks and pounds.

4) The units are fish and lakes.

5) $\approx \dfrac{5.5\text{ feet}}{1\text{ minute}}$ An increase of about five and a half feet per minute.

6) $\dfrac{-5\text{ coupons}}{1\text{ person}}$ A loss of five coupons per person.

7) $\dfrac{0.08\text{ ducks}}{1\text{ nail}}$ One nail is about eight-hundredths of a duck.

8) $\approx \dfrac{0.25\text{ mile}}{1\text{ minute}}$ An increase of about a quarter mile per minute.

9) $\dfrac{-\$4.5}{1\text{ pound}}$ A loss of four and half dollars per pound.

10) $\dfrac{-1.75\text{ pounds}}{1\text{ month}}$ A loss of three-quarter pounds per month.

11) $\dfrac{40\ \cancel{\text{nails}}}{1}\left(\dfrac{1\text{ chicken}}{10\ \cancel{\text{nails}}}\right) = 4\text{ chickens}$

12) $\dfrac{2\ \cancel{\text{pairs}}}{1}\left(\dfrac{8\text{ ducks}}{1\ \cancel{\text{pair}}}\right) = 16\text{ ducks}$

13) $\dfrac{14\ \cancel{\text{geese}}}{1}\left(\dfrac{15\text{ potatoes}}{2\ \cancel{\text{geese}}}\right) = 105\text{ potatoes}$

14) $\dfrac{7\ \cancel{\text{ducks}}}{1}\left(\dfrac{25\text{ nails}}{2\ \cancel{\text{ducks}}}\right) = \text{ about 87.5 nails}$

15) $\dfrac{75\ \cancel{\text{potatoes}}}{1}\left(\dfrac{2\text{ geese}}{15\ \cancel{\text{potatoes}}}\right) = 10\text{ geese}$

16) $\dfrac{24\ \cancel{\text{ducks}}}{1}\left(\dfrac{1\text{ pair}}{8\ \cancel{\text{ducks}}}\right) = \text{ 3 pairs of shoes}$

17) $\dfrac{12\ \cancel{\text{potatoes}}}{1}\left(\dfrac{2\ \cancel{\text{geese}}}{15\ \cancel{\text{potatoes}}}\right)\left(\dfrac{5\text{ ducks}}{1\ \cancel{\text{geese}}}\right) = 8\text{ ducks}$

18) $\dfrac{100\ \cancel{\text{nails}}}{1}\left(\dfrac{2\ \cancel{\text{ducks}}}{25\ \cancel{\text{nails}}}\right)\left(\dfrac{1\text{ pair of shoes}}{8\ \cancel{\text{ducks}}}\right) = 1\text{ pair of shoes, so yes it's possible.}$

19) $\dfrac{8\ \cancel{\text{ducks}}}{1}\left(\dfrac{25\ \cancel{\text{nails}}}{2\ \cancel{\text{ducks}}}\right)\left(\dfrac{1\text{ chicken}}{10\ \cancel{\text{nails}}}\right) = 10\text{ chickens}$

20) $\dfrac{20\ \cancel{\text{chickens}}}{1}\left(\dfrac{10\ \cancel{\text{nails}}}{1\cancel{\text{chickens}}}\right)\left(\dfrac{2\ \cancel{\text{ducks}}}{25\ \cancel{\text{nails}}}\right)\left(\dfrac{1\text{ pair of shoes}}{8\ \cancel{\text{ducks}}}\right) = 2\text{ pairs of shoes}$

21) $\dfrac{36\ \cancel{\text{chickens}}}{1}\left(\dfrac{10\ \cancel{\text{nails}}}{1\ \cancel{\text{chicken}}}\right)\left(\dfrac{2\ \cancel{\text{ducks}}}{25\ \cancel{\text{nails}}}\right)\left(\dfrac{1\ \cancel{\text{goose}}}{5\ \cancel{\text{ducks}}}\right)\left(\dfrac{15\text{ potatoes}}{2\ \cancel{\text{geese}}}\right) = \text{around 43 potatoes}$

22) $\frac{18\ \cancel{\text{potatoes}}}{1}\left(\frac{2\ \cancel{\text{geese}}}{15\ \cancel{\text{potatoes}}}\right)\left(\frac{5\ \cancel{\text{ducks}}}{1\ \cancel{\text{goose}}}\right)\left(\frac{25\ \cancel{\text{nails}}}{2\ \cancel{\text{ducks}}}\right)\left(\frac{1\ \text{chickens}}{10\ \cancel{\text{nails}}}\right) = 15\ \text{chickens}$

23) $\frac{2\ \cancel{\text{geese}}}{1}\left(\frac{5\ \cancel{\text{ducks}}}{1\ \cancel{\text{goose}}}\right)\left(\frac{25\ \cancel{\text{nails}}}{2\ \cancel{\text{ducks}}}\right)\left(\frac{1\ \text{chicken}}{10\ \cancel{\text{nails}}}\right) = \text{around } 12.5\ \text{chickens}$

24) $\frac{1\ \cancel{\text{pair}}}{1}\left(\frac{8\ \cancel{\text{ducks}}}{1\ \cancel{\text{pair}}}\right)\left(\frac{1\ \cancel{\text{goose}}}{5\ \cancel{\text{ducks}}}\right)\left(\frac{15\ \text{potatoes}}{2\ \cancel{\text{geese}}}\right) = 12\ \text{potatoes}$

1.5 Continuing with Unit Conversions

In the last section we used a barter system to practice unit conversions. Although we usually don't use barter systems anymore, many disciplines do use unit conversions.

1.5.1 Using a Conversion Table for Units of Time

One common conversion involves units for time.

Conversion Table for Time		
7 days	$\Leftrightarrow$	1 week
12 months	$\Leftrightarrow$	1 year
1 hour	$\Leftrightarrow$	60 minutes
1 year	$\Leftrightarrow$	365 days
24 hours	$\Leftrightarrow$	1 day
60 seconds	$\Leftrightarrow$	1 minute

Practice 1.5.1 Using a Conversion Table for Units of Time

Answer the following questions using unit fractions.

a) How many minutes are in a year?

$$\frac{1\text{ year}}{1}$$ $\Rightarrow$ I wrote the original amount as a fraction with a denominator of 1.

$$\frac{1\ \cancel{\text{year}}}{1}\left(\frac{\cancel{\text{days}}}{\cancel{\text{year}}}\right)\left(\frac{\cancel{\text{hours}}}{\cancel{\text{day}}}\right)\left(\frac{\text{minutes}}{\cancel{\text{hour}}}\right)$$ $\Rightarrow$ Then, I set up my fractions until the only unit left was minutes. I made sure the unit in each new denominator matched the unit in the previous numerator.

$$\frac{1\ \cancel{\text{year}}}{1}\left(\frac{365\,\cancel{\text{days}}}{1\,\cancel{\text{year}}}\right)\left(\frac{24\,\cancel{\text{hours}}}{1\,\cancel{\text{day}}}\right)\left(\frac{60\,\text{minutes}}{1\,\cancel{\text{hour}}}\right)=$$

$$\frac{1\times 365\times 24\times 60}{1\times 1\times 1\times 1}=525{,}600\text{ minutes}$$

$\Rightarrow$ To finish, I built unit fractions using numbers from the conversion table. I simplified the product and made sure to include the final unit in the answer.

Homework 1.5 Answer the following questions using unit fractions.

1) How many months are there in 30 years?

2) One day has how many seconds?

3) One year is how many hours?

4) How many days are there in 60 minutes?

5) 13 weeks is how many years?

6) 14 days is how many minutes?

7) How many years are there in 262,800 minutes?

8) 1,209,600 seconds is how many weeks?

9) How many years are in 800 weeks?

10) How many months are there in 360 hours?

1.5.2 Using a Conversion Table for English Units of Length

In the United States our current units of length are based on the English System. As you can see below, the English System was based on things like the length of your foot or the size of a piece of grain. Let's practice converting English units of length.

Conversion Table for English Units of Length		
1 Hand	⇔	4 inches
3 digits	⇔	1 nail
4 poppy seeds	⇔	1 barleycorn
1 foot	⇔	12 inches
4 digits	⇔	3 inches
1 inch	⇔	3 barleycorns

Practice 1.5.2 Using a Conversion Table for English Units of Length

Answer the following questions using unit fractions.

a) How many barleycorns are in 1 hand?

$$\frac{1\ \cancel{\text{hand}}}{1}\left(\frac{\cancel{4\ \text{inches}}}{\cancel{1\ \text{hand}}}\right)\left(\frac{3\ \text{barleycorns}}{\cancel{1\ \text{inches}}}\right)=$$

$$\frac{1\times4\times3}{1\times1\times1}=12\text{ barleycorns}$$

⇒ I found a product whose units reduced to barleycorns. Then I used the table to supply the numbers for my unit fractions. Last, I simplified making sure to include the final unit in my answer.

Homework 1.5 Answer the following questions using unit fractions.

11) 20 digits are how may feet?

12) 5 feet is how many poppy seeds?

13) How many feet is 4 nails?

14) How many hands are sixteen nails?

15) 240 poppy seeds are how many hands?

1.5.3 Converting Between English and Metric Units of Length

Instead of English units, most of the rest of the world adopted Metric units of length. Let's practice converting between some common English and Metric units.

Conversion Table for English and Metric Units of Length		
1 inch (in)	⇔	2.54 centimeters (cm)
1.61 kilometers (km)	⇔	1 mile (mi)
0.91 meters (m)	⇔	1 yard (yd)
1,000 meters (m)	⇔	1 kilometer (km)
1 yard (yd)	⇔	36 inches (in)
1 kilometer (km)	⇔	1000 meters (m)

Practice 1.5.3 Converting Between English and Metric Units of Length

Build the unit fraction(s) you need to answer the question.

a) 4 miles is how many meters?	
$\dfrac{4\ \cancel{mi}}{1}\left(\dfrac{1.61\ \cancel{km}}{1\ \cancel{mi}}\right)\left(\dfrac{1{,}000\ m}{1\ \cancel{km}}\right) \approx 6{,}440$ meters $\Rightarrow$	I built the unit fractions, reduced units and multiplied.

Homework 1.5 Build the unit fraction(s) you need to answer the question.

16) 50 centimeters is how many yards?

17) How many miles is 400 meters?

18) How many meters is 72 inches?

19) 19,685 inches is how many kilometers?

20) 1 mile is how many yards?

Homework 1.5

1) $$\frac{30\ \cancel{\text{years}}}{1}\left(\frac{12\text{ months}}{1\ \cancel{\text{year}}}\right)=360\text{ months}$$

2) $$\frac{1\ \cancel{\text{day}}}{1}\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\right)\left(\frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}}\right)\left(\frac{60\text{ seconds}}{1\ \cancel{\text{minute}}}\right)=86,400\text{ seconds}$$

3) $$\frac{1\ \cancel{\text{year}}}{1}\left(\frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}}\right)\left(\frac{24\text{ hours}}{1\ \cancel{\text{days}}}\right)=8,760\text{ hours}$$

4) $$\frac{60\ \cancel{\text{minutes}}}{1}\left(\frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}}\right)\left(\frac{1\text{ day}}{24\ \cancel{\text{hours}}}\right)=\frac{1}{24}\text{ day}$$

5) $$\frac{13\ \cancel{\text{weeks}}}{1}\left(\frac{7\ \cancel{\text{days}}}{1\ \cancel{\text{week}}}\right)\left(\frac{1\text{ year}}{365\ \cancel{\text{days}}}\right)\approx 0.25\text{ or }\frac{1}{4}\text{ year}$$

6) $$\frac{14\ \cancel{\text{days}}}{1}\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{days}}}\right)\left(\frac{60\text{ minutes}}{1\ \cancel{\text{hour}}}\right)=20{,}160\text{ minutes}$$

7) $$\frac{262,800\ \cancel{\text{minutes}}}{1}\left(\frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}}\right)\left(\frac{1\ \cancel{\text{day}}}{24\ \cancel{\text{hours}}}\right)\left(\frac{1\text{ year}}{365\ \cancel{\text{days}}}\right)=\frac{1}{2}\text{ year}$$

8) $$\frac{1,209,600\ \cancel{\text{seconds}}}{1}\left(\frac{1\ \cancel{\text{minute}}}{60\ \cancel{\text{seconds}}}\right)\left(\frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}}\right)\left(\frac{1\ \cancel{\text{day}}}{24\ \cancel{\text{hours}}}\right)\left(\frac{1\text{ week}}{7\ \cancel{\text{days}}}\right)=2\text{ weeks}$$

9) $$\frac{800\ \cancel{\text{weeks}}}{1}\left(\frac{7\ \cancel{\text{days}}}{1\ \cancel{\text{week}}}\right)\left(\frac{1\text{ year}}{365\ \cancel{\text{days}}}\right)=15.34\text{ years}$$

10) $$\frac{360\ \cancel{\text{hours}}}{1}\left(\frac{1\ \cancel{\text{day}}}{24\ \cancel{\text{hours}}}\right)\left(\frac{1\ \cancel{\text{year}}}{365\ \cancel{\text{days}}}\right)\left(\frac{12\text{ months}}{1\ \cancel{\text{year}}}\right)\approx\text{ half a month}$$

11) $$\frac{20\ \cancel{\text{digits}}}{1}\left(\frac{3\ \cancel{\text{inches}}}{4\ \cancel{\text{digits}}}\right)\left(\frac{1\text{ foot}}{12\ \cancel{\text{inchs}}}\right)=1.25\text{ feet}$$

12) $$\frac{5\ \cancel{\text{feet}}}{1}\left(\frac{12\ \cancel{\text{inches}}}{1\ \cancel{\text{foot}}}\right)\left(\frac{3\ \cancel{\text{barleycorns}}}{1\ \cancel{\text{inch}}}\right)\left(\frac{4\text{ poppy seeds}}{1\ \cancel{\text{barleycorns}}}\right)=720\text{ poppy seeds}$$

13) $$\frac{4\ \cancel{\text{nails}}}{1}\left(\frac{3\ \cancel{\text{digits}}}{1\ \cancel{\text{nail}}}\right)\left(\frac{3\ \cancel{\text{inches}}}{4\ \cancel{\text{digits}}}\right)\left(\frac{1\text{ foot}}{12\ \cancel{\text{inches}}}\right)=0.75\text{ feet}$$

14) $$\frac{16\ \cancel{\text{nails}}}{1}\left(\frac{3\ \cancel{\text{digits}}}{1\ \cancel{\text{nail}}}\right)\left(\frac{3\ \cancel{\text{inches}}}{4\ \cancel{\text{digits}}}\right)\left(\frac{1\text{ hand}}{4\ \cancel{\text{inches}}}\right)=9\text{ hands}$$

15) $\frac{240\ \cancel{\text{poppy seeds}}}{1}\left(\frac{1\ \cancel{\text{barleycorns}}}{4\ \cancel{\text{poppy seeds}}}\right)\left(\frac{1\ \cancel{\text{inch}}}{3\ \cancel{\text{barleycorns}}}\right)\left(\frac{1\ \text{hand}}{4\ \cancel{\text{inches}}}\right) = 5\ \text{hands}$

16) $\frac{50\ \cancel{\text{cm}}}{1}\left(\frac{1\ \cancel{\text{in}}}{2.54\ \cancel{\text{cm}}}\right)\left(\frac{1\ \text{yd}}{36\ \cancel{\text{in}}}\right) \approx 0.55\ \text{yd}$

17) $\frac{400\ \cancel{\text{m}}}{1}\left(\frac{1\ \cancel{\text{km}}}{1{,}000\ \cancel{\text{m}}}\right)\left(\frac{1\ \text{mi}}{1.61\ \cancel{\text{km}}}\right) \approx 0.25\ \text{miles}$

18) $\frac{72\ \cancel{\text{in}}}{1}\left(\frac{1\ \cancel{\text{yd}}}{36\ \cancel{\text{in}}}\right)\left(\frac{0.91\ \text{m}}{1\ \cancel{\text{yd}}}\right) \approx 1.8\ \text{meters}$

19) $\frac{19{,}685\ \cancel{\text{in}}}{1}\left(\frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}}\right)\left(\frac{1\ \cancel{\text{m}}}{100\ \cancel{\text{cm}}}\right)\left(\frac{1\ \text{km}}{1{,}000\ \cancel{\text{m}}}\right) \approx 0.5\ \text{km}$

20) $\frac{1\ \cancel{\text{mi}}}{1}\left(\frac{1.61\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1{,}000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \text{yd}}{0.91\ \cancel{\text{m}}}\right) \approx 1{,}769\ \text{yd}$

1.6 Two-Item Unit Conversions

So far, only our numerator unit has needed to change. In this lesson both our numerator unit and our denominator unit will need to change. Here are the unit conversions we'll be using.

English./Metric Units of Length	Units of Time	English./Metric Units of Volume
1 foot ⇔ 12 inches	60 seconds ⇔ 1 minute	3 teaspoons ⇔ 1 tablespoon
1,000 meters ⇔ 1 kilometer	60 minutes ⇔ 1hour	1 cup ⇔ 16 tablespoons
5,280 feet ⇔ 1 mile	1 week ⇔ 7 days	16 cups ⇔ 1 gallon
1,609 meters ⇔ 1 mile	365 days ⇔ 1 year	1 gallon ⇔ 3.8 liters
100 centimeters ⇔ 1 meter	1 day ⇔ 24 hours	14.8 milliliters ⇔ 1 tablespoon

1.6.1 Beginning Two-Item Unit Conversions

In practice it's common to convert both a numerator and denominator unit within the same problem. A good strategy for two-item problems is to write your original rate, including the units, and then to the right write your final rate using only units. Next, put together a chain of fractions, from left to right, in a way that all units reduce except those you need for your final rate. Last, supply numbers to build unit fractions, multiply and reduce to answer the question.

For example, say I wanted to covert 200 feet per second to miles per hour. I've began the problem to the right.

$$\frac{200\text{ feet}}{1\text{ second}} \qquad \frac{\text{miles}}{\text{hour}}$$

Next, I notice that using the table I'm able to go from feet to miles (my final numerator unit). After reducing I'm left with miles per second.

$$\frac{200\ \cancel{\text{feet}}}{1\text{ second}}\left(\frac{\text{mile}}{\cancel{\text{feet}}}\right) \qquad \frac{\text{miles}}{\text{hour}}$$

I returned to my original rate and try to convert seconds to hours, but the conversion table only has seconds to minutes. Notice that after reducing seconds I have miles per minute.

$$\frac{200\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}}\left(\frac{\text{mile}}{\cancel{\text{feet}}}\right)\left(\frac{\cancel{\text{seconds}}}{\text{minute}}\right) \qquad \frac{\text{miles}}{\text{hour}}$$

Using the table, I notice I can go from minutes to hours. After reducing, the units left are miles per hour.

$$\frac{200\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}}\left(\frac{\text{mile}}{\cancel{\text{feet}}}\right)\left(\frac{\cancel{\text{seconds}}}{\cancel{\text{minute}}}\right)\left(\frac{\cancel{\text{minutes}}}{\text{hour}}\right) \qquad \frac{\text{miles}}{\text{hour}}$$

Now, using the table, I can fill in the numbers and finish the problem to find that 200 feet per second is approximately 136.4 miles per hour.

$$\frac{200\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}}\left(\frac{1\text{ mile}}{5{,}280\ \cancel{\text{feet}}}\right)\left(\frac{60\ \cancel{\text{seconds}}}{1\ \cancel{\text{minute}}}\right)\left(\frac{60\ \cancel{\text{minutes}}}{1\text{ hour}}\right)$$

$$\frac{200(60)(60)}{5{,}280} \approx \frac{136.4\text{ miles}}{\text{hour}}$$

Practice 1.6.1 Beginning Two-Item Unit Conversions
Use unit conversions to answer the question.

a) Convert 2 cups per centimeter to liters per mile.

$\frac{2\text{ cups}}{1\text{ cm}} \qquad \frac{\text{liters}}{\text{mile}}$ ⇒ I wrote the original rate and the units of the rate I'm after.

$\frac{2\ \cancel{\text{cups}}}{1\text{ cm}}\left(\frac{\text{gallon}}{\cancel{\text{cups}}}\right) \qquad \frac{\text{liters}}{\text{mile}}$ ⇒ I began by converting cups to gallons.

$\frac{2\ \cancel{\text{cups}}}{1\text{ cm}}\left(\frac{\cancel{\text{gallon}}}{\cancel{\text{cups}}}\right)\left(\frac{\text{liters}}{\cancel{\text{gallons}}}\right) \qquad \frac{\text{liters}}{\text{mile}}$ ⇒ Next I converted gallons to liters. My numerator unit is now finished.

$\frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{cm}}}\left(\frac{\cancel{\text{gallon}}}{\cancel{\text{cups}}}\right)\left(\frac{\text{liters}}{\cancel{\text{gallons}}}\right)\left(\frac{\cancel{\text{cm}}}{\text{m}}\right) \qquad \frac{\text{liters}}{\text{mile}}$ ⇒ I started back at the original and converted centimeters to meters. I now have liters per meter.

$\frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{cm}}}\left(\frac{\cancel{\text{gallon}}}{\cancel{\text{cups}}}\right)\left(\frac{\text{liters}}{\cancel{\text{gallons}}}\right)\left(\frac{\cancel{\text{cm}}}{\cancel{\text{m}}}\right)\left(\frac{\cancel{\text{m}}}{\text{mile}}\right) \qquad \frac{\text{liters}}{\text{mile}}$ ⇒ I converted meters to miles. My units now match the final units.

$\frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{cm}}}\left(\frac{1\ \cancel{\text{gallon}}}{16\ \cancel{\text{cups}}}\right)\left(\frac{3.8\text{ liters}}{1\ \cancel{\text{gallons}}}\right)\left(\frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}}\right)\left(\frac{1{,}609\ \cancel{\text{m}}}{1\text{ mile}}\right)$

$\frac{2(3.8)(100)(1{,}609)}{16} = \frac{76{,}427.5\text{ liters}}{\text{mile}}$ ⇒ I supplied the numbers using the tables and finished the problem.

Homework 1.6 Use unit conversions to answer the question.

1) Covert 55 miles per hour to feet per minute.

2) Covert 400 meters per second to kilometers per minute.

3) Covert $1,775 per year to dollars per week.

4) Covert 7 miles in 5 minutes to kilometers per hour.

5) Convert 100 kilometers per 18 liters to miles per gallon?

6) The speed of light is 300,000 km per second. How many miles per hour is that?

1.6.2 Applying Two-Item Unit Conversions

Now let's use unit conversions to answer some applied questions.

Practice 1.6.2 Applying Two-Item Unit Conversions
Build the unit fraction(s) you need to answer the question.

a) A Boeing 747 burns about 250 liters of fuel per minute. If it takes 3.5 hours to fly the 1,400 miles from MSP (Minneapolis/St. Paul International Airport) to RSW (Southwest Florida International Airport) what is the fuel use rate in gallons per mile? Round to the nearest gallon. (Please notice this isn't miles per gallon).

$\frac{250 \text{ liters}}{1 \text{ minute}} \qquad \left(\frac{\text{gallons}}{\text{mile}}\right)$	$\Rightarrow$	I wrote the original rate and the final rate that I'm looking for.
$\frac{250 \cancel{\text{liters}}}{1 \text{ minute}}\left(\frac{\text{gallon}}{\cancel{\text{liters}}}\right)$	$\Rightarrow$	I noticed the table showed how to go from liters to gallons, so I started with that. I now have my final numerator unit of gallons.
$\frac{250 \cancel{\text{liters}}}{1 \cancel{\text{minute}}}\left(\frac{\text{gallon}}{\cancel{\text{liters}}}\right)\left(\frac{\cancel{\text{minutes}}}{\text{hour}}\right)$	$\Rightarrow$	I decided to convert minutes to hours since a lot of the original information in the problem is in hours. My rate is now gallons per hour, but I need the rate gallons per mile so I need a way to convert hours to miles.
If it takes 3.5 hours to fly the 1,400 miles from MSP to RSW.	$\Rightarrow$	I couldn't find a conversion from gallons to miles in the table, so I went back to the original information and noticed there was a relationship between hours and miles.
$\frac{250 \cancel{\text{liters}}}{1 \cancel{\text{minute}}}\left(\frac{\text{gallon}}{\cancel{\text{liters}}}\right)\left(\frac{\cancel{\text{minutes}}}{\cancel{\text{hour}}}\right)\left(\frac{\cancel{\text{hours}}}{\text{miles}}\right)$	$\Rightarrow$	I'm now able to go from hours to miles. I have my final units of gallons per mile.
$\frac{250 \cancel{\text{liters}}}{1 \cancel{\text{minute}}}\left(\frac{1 \text{ gallon}}{3.8 \cancel{\text{liters}}}\right)\left(\frac{60 \cancel{\text{minutes}}}{1 \cancel{\text{hour}}}\right)\left(\frac{3.5 \cancel{\text{hours}}}{1{,}400 \text{ miles}}\right)$	$\Rightarrow$	I filled in the numbers to build my unit fractions.
$\frac{250(60)(3.5) \text{ gallons}}{3.8(1{,}400) \text{ miles}} \approx \frac{10 \text{ gallons}}{\text{mile}}$	$\Rightarrow$	I multiplied and reduced making sure to keep my final units. The airplane uses about 10 gallons of fuel to fly one mile.

Homework 1.6 Build the unit fraction(s) you need to answer the question.

7) If the sun is 93,000,000 miles from the Earth how many minutes will it take for light from the sun to reach the Earth.? (You might need your answer from question 6.)

8) You have a leaky faucet in the basement that drips half a cup of water per hour. If you don't fix the faucet for a year, how many gallons of water have been wasted?

9) In baseball the average fastball travels about 140 feet in a second. How fast (in miles per hour) would you have to be driving a car to keep up with a fastball?

10) While camping you need to give your child medicine. The dosage on the new 250-milliliter bottle says you should give 2 tablespoons every 12 hours. If you'll be giving your child two doses today, and then camping for 3 more days (for a total of four days), will you have enough medicine?

11) A cruise ship that's traveling 3,400 miles from England to New York burns 850 gallons of fuel an hour. If the trip takes 8 days, what is the ships gallons per mile? (Please notice this is gallons per mile not miles per gallon.)

12) Suppose you want to hike a 7-mile portion of the Superior Hiking Trail in North East MN. At home, you pace yourself and find out that you walk at about 250 feet per minute. How many hours will it take for you to hike the 7 miles?

13) The average person drives 40 miles a day and the average car has a fuel economy of 25.4 mpg (miles per gallon). If gas is approximately $2.45 per gallon estimate the cost an average person spends per year on gasoline.

14) Estimate the annual (yearly) revenue of a fast food restaurant. Suppose that the average revenue per person is $4.50 and that the restaurant serves an average of 3,500 people each week.

15) You are throwing a pizza party for 18 people and figure each person might eat 4 slices of pizza. The pizza place charges $14 per pizza and it will be cut into 12 slices. What will the cost of the pizza be?

16) A "running" toilet can often be fixed for under $5. Typically, a running toilet will waste 8 gallons of water per hour. If the cost of water in Minneapolis is $40.49 for 7,500 gallons how much will a running toilet cost you if you let it go for a year?

Homework 1.6

1) $\frac{55\ \cancel{\text{miles}}}{1\ \cancel{\text{hour}}}\left(\frac{5{,}280\text{ feet}}{1\ \cancel{\text{mile}}}\right)\left(\frac{1\ \cancel{\text{hour}}}{60\text{ minutes}}\right)=\frac{4{,}840\text{ feet}}{\text{minute}}$

2) $\frac{400\ \cancel{\text{meters}}}{1\ \cancel{\text{second}}}\left(\frac{60\ \cancel{\text{seconds}}}{1\text{ minute}}\right)\left(\frac{1\text{ kilometer}}{1{,}000\ \cancel{\text{meters}}}\right)=\frac{24\text{ kilometers}}{\text{minute}}$

3) $\frac{\$1{,}775}{1\ \cancel{\text{year}}}\left(\frac{1\ \cancel{\text{year}}}{365\ \cancel{\text{days}}}\right)\left(\frac{7\ \cancel{\text{days}}}{1\text{ week}}\right)\approx\frac{\$34}{\text{week}}$

4) $\frac{7\ \cancel{\text{miles}}}{5\ \cancel{\text{minutes}}}\left(\frac{1{,}609\ \cancel{\text{meters}}}{1\ \cancel{\text{mile}}}\right)\left(\frac{1\text{ kilometer}}{1{,}000\ \cancel{\text{meters}}}\right)\left(\frac{60\ \cancel{\text{minutes}}}{1\text{ hour}}\right)\approx\frac{135\text{ kilometers}}{\text{hour}}$

5) $\frac{100\ \cancel{\text{kilometers}}}{18\ \cancel{\text{liters}}}\left(\frac{1{,}000\ \cancel{\text{meters}}}{1\ \cancel{\text{kilometer}}}\right)\left(\frac{1\text{ mile}}{1{,}609\ \cancel{\text{meters}}}\right)\left(\frac{3.8\ \cancel{\text{liters}}}{1\text{ gallon}}\right)\approx\frac{13.1\text{ miles}}{\text{gallon}}$

6) $\frac{300{,}000\ \cancel{\text{km}}}{1\ \cancel{\text{sec}}}\left(\frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{mn}}}\right)\left(\frac{60\ \cancel{\text{mn}}}{1\text{ hour}}\right)\left(\frac{1{,}000\ \cancel{\text{meters}}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\text{ mile}}{1{,}609\ \cancel{\text{meters}}}\right)\approx\frac{671{,}224{,}363\text{ miles}}{\text{hour}}$

7) $\frac{1\ \cancel{\text{hour}}}{671{,}224{,}363\ \cancel{\text{miles}}}\left(\frac{93{,}000{,}000\ \cancel{\text{miles}}}{1}\right)\left(\frac{60\text{ minutes}}{1\ \cancel{\text{hour}}}\right)\approx 8.3\text{ minutes}$

8) $\frac{0.5\ \cancel{\text{cups}}}{1\ \cancel{\text{hour}}}\left(\frac{1\text{ gallon}}{16\ \cancel{\text{cups}}}\right)\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\right)\left(\frac{365\ \cancel{\text{days}}}{1\text{ year}}\right)\approx\frac{273.75\text{ gallons}}{\text{year}}$

9) $\frac{140\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}}\left(\frac{1\text{ mile}}{5{,}280\ \cancel{\text{feet}}}\right)\left(\frac{60\ \cancel{\text{seconds}}}{1\ \cancel{\text{minute}}}\right)\left(\frac{60\ \cancel{\text{minutes}}}{1\text{ hour}}\right)\approx 95.5\text{ miles per hour}$

10) $\frac{2\ \cancel{\text{tablespoons}}}{12\ \cancel{\text{hours}}}\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\right)\left(\frac{4\ \cancel{\text{days}}}{1}\right)\left(\frac{14.8\text{ milliliters}}{1\ \cancel{\text{tablespoon}}}\right)=\ 236\text{ milliliters}$ Yes you should.

11) $\frac{850\text{ gallons}}{1\ \cancel{\text{hour}}}\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\right)\left(\frac{8\ \cancel{\text{days}}}{3{,}400\text{ miles}}\right)=\frac{48\text{ gallons}}{1\text{ mile}}$

12) $\frac{1\ \cancel{\text{minute}}}{250\ \cancel{\text{feet}}}\left(\frac{5{,}280\ \cancel{\text{feet}}}{1\ \cancel{\text{mile}}}\right)\left(\frac{7\ \cancel{\text{miles}}}{1}\right)\left(\frac{1\text{ hour}}{60\ \cancel{\text{minutes}}}\right)\approx 2.5\text{ hours}$

13) $\frac{40\ \cancel{\text{miles}}}{1\ \cancel{\text{day}}}\left(\frac{365\ \cancel{\text{days}}}{1\text{ year}}\right)\left(\frac{1\ \cancel{\text{gallon}}}{25.4\ \cancel{\text{miles}}}\right)\left(\frac{\$2.45}{1\ \cancel{\text{gallon}}}\right)\approx\frac{\$1{,}408}{\text{year}}$

14) $\frac{\$4.50}{1\ \cancel{\text{person}}}\left(\frac{3{,}500\ \cancel{\text{people}}}{1\ \cancel{\text{week}}}\right)\left(\frac{1\ \cancel{\text{week}}}{7\ \cancel{\text{days}}}\right)\left(\frac{365\ \cancel{\text{days}}}{1\text{ year}}\right)=\frac{\$821{,}250}{\text{year}}$

15) $\frac{\$14}{12\ \cancel{\text{slices}}}\left(\frac{4\ \cancel{\text{slices}}}{1\ \cancel{\text{person}}}\right)\left(\frac{18\ \cancel{\text{people}}}{1}\right)=\$84$

16) $\frac{8\ \cancel{\text{gallons}}}{1\ \cancel{\text{hour}}}\left(\frac{\$40.49}{7{,}500\ \cancel{\text{gallons}}}\right)\left(\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\right)\left(\frac{365\ \cancel{\text{days}}}{1\text{ year}}\right)\approx\frac{\$378}{\text{year}}$

1.7 Proportions

If you set two rates equal, you have a proportion. Proportions are useful when we know both the numerator and denominator of one rate and either the numerator or the denominator of the other. In this case, we're able to solve for the value of the unknown numerator or denominator in the second rate. Here's the procedure we'll use to solve a proportion.

Procedure Solving a Proportion
1. Identify the unit of your question.
2. On the left, build a rate. The unit of your question goes in the numerator with an *x* in place of the missing value. Identify the value and unit that goes with it and put these in the denominator.
3. On the right, build a rate using the known values and units. The units in both numerators should match and the units in both denominators should match.
4. Multiply both rates by the value in the denominator of the rate on the left and simplify both expressions to solve the proportion.
5. Make sure you answer the question using the correct unit.

1.7.1 Solving a Proportion for an Unknown Value

Before we begin applying proportions, let's make sure you're comfortable with step 4 of the procedure where you solve a proportion equation.

Practice 1.7.1 Solving a Proportion for an Unknown Value

Solve for the unknown value.

a) $\frac{x}{14} = \frac{9}{7}$

$\left(\frac{14}{1}\right)\left(\frac{x}{14}\right) = \left(\frac{9}{7}\right)\left(\frac{14}{1}\right)$	$\Rightarrow$	I multiplied both sides by the value of the left denominator.
$\left(\frac{\cancel{14}}{1}\right)\left(\frac{x}{\cancel{14}}\right) = \left(\frac{9}{7}\right)\left(\frac{14}{1}\right)$ $x = \left(\frac{9}{7}\right)\left(\frac{14}{1}\right)$	$\Rightarrow$	On the left I reduced the common factor of 14.
$x = 18$	$\Rightarrow$	Then I simplified the expression on the right. The unknown value is 18.

b) $\dfrac{x}{1\frac{1}{2}} = \dfrac{3\frac{1}{4}}{8}$	
$\dfrac{x}{1.5} = \dfrac{3.25}{8}$	$\Rightarrow$ I wrote the mixed numbers as decimals.
$\left(\dfrac{1.5}{1}\right)\left(\dfrac{x}{1.5}\right) = \left(\dfrac{3.25}{8}\right)\left(\dfrac{1.5}{1}\right)$	$\Rightarrow$ Then I multiplied both sides by the denominator of the rate on the left.
$\left(\dfrac{\cancel{1.5}}{1}\right)\left(\dfrac{x}{\cancel{1.5}}\right) = \left(\dfrac{3.25}{8}\right)\left(\dfrac{1.5}{1}\right)$ $x \approx 0.61$	$\Rightarrow$ On the left I reduced the common factor of 1.5 and on the right I simplified. The unknown value is about 0.61

Homework 1.7 Solve for the unknown value.

1) $\dfrac{x}{41} = \dfrac{66}{11}$	2) $\dfrac{x}{270} = \dfrac{8}{20}$	3) $\dfrac{x}{3.5} = \dfrac{12}{0.7}$	4) $\dfrac{x}{4} = \dfrac{4{,}050}{1{,}200}$
5) $\dfrac{x}{12} = \dfrac{1{,}480}{8}$	6) $\dfrac{x}{0.95} = \dfrac{1.05}{1.33}$	7) $\dfrac{x}{3\frac{1}{4}} = \dfrac{12\frac{3}{4}}{6.63}$	8) $\dfrac{x}{4\frac{1}{4}} = \dfrac{1}{0.017}$

1.7.2 Applying Proportions

Now let's use a proportion to answer a question. Say we'd just completed a trip where we'd traveled 210 miles on 15 gallons of gas. If our next trip was 700 miles, how much gas should we expect to use? Following the procedure, I'd first identify that our question is about gallons of gas, so I'd build my rate on the left with *x* gallons in the numerator. Since 700 miles goes with this numerator, I'd put that information in the denominator, $\dfrac{x \text{ gallons}}{700 \text{ miles}} =$. Now, on the right, I'd build a second rate with gallons in the numerator and miles in the denominator so the units in the numerators and denominators match, $\dfrac{x \text{ gallons}}{700 \text{ miles}} = \dfrac{\text{gallons}}{\text{miles}}$. At the same time, I'd supply the numbers we have from the first trip, $\dfrac{x \text{ gallons}}{700 \text{ miles}} = \dfrac{15 \text{ gallons}}{210 \text{ miles}}$. Although you can solve the proportion now (step 4 in the procedure), I like to rewrite the proportion without units first $\left(\dfrac{x}{700}\right) = \left(\dfrac{15}{210}\right)$ and then complete step 4, $\left(\dfrac{\cancel{700}}{1}\right)\left(\dfrac{x}{\cancel{700}}\right) = \left(\dfrac{15}{210}\right)\left(\dfrac{700}{1}\right)$, $x = 50$. We estimate it will take about 50 gallons of gas to make the 700 mile trip. Here's some practice.

Practice 1.7.2 Applying Proportions

Build and use a proportion to answer the question.

a) How long will it take to read 200 pages if it took an hour and a half to read 60 pages?

$$\frac{x \text{ hours}}{200 \text{ pages}} =$$

$\Rightarrow$ The question is asking how long it will take so hours is the unit in the numerator on the left. In the left denominator I put the 200 pages that goes with it.

$$\frac{x \text{ hours}}{200 \text{ pages}} = \frac{1.5 \text{ hours}}{60 \text{ pages}}$$

$\Rightarrow$ On the right I put the information for the rate where both values are known. I made sure to match the units.

$$\left(\frac{200}{1}\right)\left(\frac{x}{200}\right) = \left(\frac{1.5}{60}\right)\left(\frac{200}{1}\right)$$

$\Rightarrow$ I multiplied both expressions by the value of the left denominator.

$$\left(\frac{\cancel{200}}{1}\right)\left(\frac{x}{\cancel{200}}\right) = \left(\frac{1.5}{60}\right)\left(\frac{200}{1}\right)$$

$$x = 5$$

$\Rightarrow$ On the left I reduced the common factor. On the right I simplified. It will take about 5 hours to finish the 200 pages.

b) If taxes on a \$350,000 house are \$4,850, estimate the taxes on a \$220,000 house. Round your answer to the nearest dollar.

$$\frac{\$x \text{ tax}}{\$220{,}000 \text{ house}} =$$

$\Rightarrow$ The question is about the taxes, so tax dollars is the unit in the numerator on the left. In the denominator on the left I'd put the \$220,000 house value that goes with it.

$$\frac{\$x \text{ tax}}{\$220{,}000 \text{ house}} = \frac{\$4{,}850 \text{ tax}}{\$350{,}000 \text{ house}}$$

$\Rightarrow$ On the right I'd put the information for the rate where both values are known. I made sure to match the units on the left and right.

$$\left(\frac{220{,}000}{1}\right)\left(\frac{x}{220{,}000}\right) = \left(\frac{4{,}850}{350{,}000}\right)\left(\frac{220{,}000}{1}\right)$$

$\Rightarrow$ I multiplied both expressions by the value of the left denominator.

$$\left(\frac{\cancel{220{,}000}}{1}\right)\left(\frac{x}{\cancel{220{,}000}}\right) = \left(\frac{4{,}850}{350{,}000}\right)\left(\frac{220{,}000}{1}\right)$$

$$x = \$3{,}049$$

$\Rightarrow$ I reduced the common factor on the left and simplified the expression on the right. The taxes will be \$3,049.

Homework 1.7 Use unit conversions to answer the question.

9) Estimate a person's weight loss after 30 weeks if, after changing their diet for 12 weeks, the person had lost 8 pounds.

10) Find the fine for someone going 25 mph over the speed limit if someone going 8 mph over the speed limit had to pay a \$96 fine.

11) After washing 11 cars a high school car wash has earned a profit of \$66. After washing 41 cars what will the profit be?

12) Estimate the number of people who will have voted after 2 hours if after 20 minutes 18 people have voted.

13) Proportions can be used to estimate animal populations. Within a 20 acre parcel of a 270 acre park 8 deer are counted. Estimate the number of deer in the entire park.

14) After 2 years \$10 in simple interest has been earned on an investment. After 5 years how much will have been earned in simple interest?

15) Estimate how long it will take for all 210 employees to exit the building if it takes half a minute for the first 15 employees to exit the building during a fire drill.

16) A recipe calls for $\frac{1}{4}$ cup of vinegar for every $\frac{1}{2}$ cup of sugar. If $2\frac{1}{2}$ cups of vinegar are used how much sugar should be added to the recipe?

17) A student is paying \$1,480 for 8 credits. Estimate their cost if they decide to take 12 credits.

Homework 1.7 Answers

1) 246	2) 108	3) 60	4) 13.5	5) 2,220	6) 0.75
7) 0.1625	8) 250	9) 20 pounds	10) \$300	11) \$246	12) 108 people
13) 108 deer	14) \$25	15) 7 minutes	16) 5 cups	17) \$2,220	